

48th Annual Dick Schaff Math Superbowl

2023 Level 5 Exam – Secondary Math III Blitz

- Directions:
1. Select the most correct answer for each question and mark it on your Scantron.
 2. NO CALCULATORS OR ELECTRONIC DEVICES MAY BE USED.
 3. Note that N.O.T. means “none of these.”

1) On an essay in an English class, the median score was 75, the mean score was 68, and the standard deviation was 6. To curve the essay scores, the teacher adds 8 points to each person’s exam score. Which of the following is true?

- A. The new mean score is 68, and the median is 75, and the new standard deviation is 14.
- B. The new mean score is 68, and the median is 83, and the new standard deviation is 14.
- C. The new mean score is 76, and the median is 75, and the new standard deviation is 6.
- D. The new mean score is 76, and the median is 83, and the new standard deviation is 6.
- E. N.O.T.

2) The domain of the function $f(x) = \sqrt{\frac{2x-1}{x+7}}$ is:

- A. $(-\infty, -7] \cup \left[\frac{1}{2}, \infty\right)$
- B. $(-\infty, -7] \cup \left(\frac{1}{2}, \infty\right)$
- C. $(-\infty, -7) \cup \left[\frac{1}{2}, \infty\right)$
- D. $(-\infty, -7) \cup \left(\frac{1}{2}, \infty\right)$
- E. N. O. T.

3) Which of the following polynomials has roots i , $-i$, -2 , and 5 ?

- A. $x^4 - 3x^3 - 9x^2 - 3x - 10$
- B. $x^4 - 3x^3 - 11x^2 + 3x + 10$
- C. $x^4 + 3x^3 - 9x^2 + 3x - 10$
- D. $x^4 - 3x^3 + 11x^2 - 3x - 10$
- E. N. O. T.

4) Which of the following statements is true about the graph of $g(x) = (x + 3)^2 + 5$?

- A. The graph of $g(x)$ is obtained by shifting the graph of $f(x) = x^2$ up 5 units and right 3 units.
- B. The graph of $g(x)$ is obtained by shifting the graph of $f(x) = x^2$ down 5 units and right 3 units.
- C. The graph of $g(x)$ is obtained by shifting the graph of $f(x) = x^2$ up 5 units and left 3 units.
- D. The graph of $g(x)$ is obtained by shifting the graph of $f(x) = x^2$ down 5 units and left 3 units.
- E. N. O. T.

5) Which of the following equations is equivalent to the equation $\log_4(x) - 7 = 2 \log_4(z)$?

- A. $4x - 7 = 4z^2$
- B. $xz^2 = 4^7$
- C. $x^4 - 7 = 2z^4$
- D. $\frac{x}{z^2} = 4^7$
- E. N.O. T.

6) If $x + 3$ is a factor of $x^3 - 2x^2 + kx + 27$, then what must the value of k be?

- A. 3
- B. -3
- C. 6
- D. -6
- E. N.O. T.

7) A sequence is defined recursively by $a_n = 3a_{n-1} - a_{n-2}$ for $n \geq 3$. If $a_1 = 1$ and $a_2 = 1$, find a_7 .

- A. 178
- B. 89
- C. 34
- D. 13
- E. N. O. T.

8) Let $f(x) = \frac{x^2 - 5x + 4}{x - 4}$ and $g(x) = x - 1$. Which of the following statements is true?

- A. The graphs of $f(x)$ and $g(x)$ have no points in common.
- B. The graphs of $f(x)$ and $g(x)$ are exactly the same.
- C. The graphs of $f(x)$ and $g(x)$ are the same, except at $x = 4$.
- D. The graphs of $f(x)$ and $g(x)$ are the same, except at $x = 1$.
- E. N.O.T.

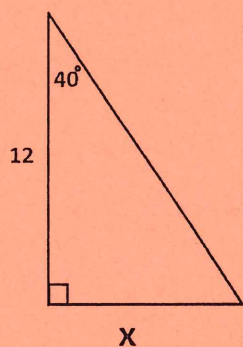
9) A normally distributed data set has a particular data value with a z-score of $z = 0.9$. Which of the following describes the meaning of this z-score?

- A. This particular data value is 0.9 standard deviations below the median.
- B. This particular data value is 0.9 standard deviations below the mean.
- C. This particular data value is 0.9 standard deviations above the median.
- D. This particular data value is 0.9 standard deviations above the mean.
- E. N.O.T.

10) If $f(x) = 2x^2 - 3x$, which of the following represents $\frac{f(x+h)-f(x)}{h}$?

- A. $4xh + 2h^2 - 3h$
- B. $2xh + 2h^2 - 3h - 6x$
- C. $4xh - 3h$
- D. $2xh + 2h^2 - 3h$
- E. N.O.T.

11) For the triangle pictured below, what is the exact value of x ?



- A. $12 \sin(40^\circ)$
- B. $12 \cos(40^\circ)$
- C. $12 \tan(40^\circ)$
- D. $\frac{\sin(40^\circ)}{12}$
- E. N.O.T.

12) What is the coefficient of x^4y^2 in the expansion of $(2x + y)^6$?

- A. 320
- B. 160
- C. 60
- D. 15
- E. N.O.T.

13) What is the solution set to the equation $x + 2\sqrt{3 - x} = 0$?

- A. $\{0\}$ B. $\{-6, 2\}$ C. $\{-6\}$ D. $\{-2\}$ E. N.O.T.

14) A group of psychologists wants to find the average age that a college student in the United States graduates with a bachelor's degree. They survey 400 randomly selected college graduates throughout the country and ask them how old they were when they earned their bachelor's degree. Which group of students represents the sample?

- A. Everyone in the United States.
B. All college graduates with a bachelor's degree in the United States.
C. All college graduates and those currently in college in the United States.
D. The 400 college students surveyed.
E. N.O.T.

15) Solve for x : $5e^{x-2} = 12$

- A. $x = \ln\left(\frac{12}{5}\right) + 2$ B. $x = \ln\left(\frac{12}{5}\right) - 2$
C. $x = \ln(7) + 2$ D. $x = \ln\left(\frac{5}{12}\right)$ E. N.O.T.

16) Find the solution set for: $4x^2 + 5x - 6 \geq 0$

- A. $\left(-2, \frac{3}{4}\right)$ B. $\left[-2, \frac{3}{4}\right]$
C. $(-\infty, -2) \cup \left(\frac{3}{4}, \infty\right)$ D. $(-\infty, -2] \cup \left[\frac{3}{4}, \infty\right)$ E. N.O.T.

17) Find the radius of a circle if the central angle of $\frac{4\pi}{3}$ subtends an arclength of 24π .

- A. $\frac{96\pi}{3}$ B. 6π C. 18 D. 32 E. N. O. T.

18) A region is bounded by the y -axis, the x -axis, the line $y = 4$, and the line $x = 2$. What three-dimensional shape is formed when you rotate the bounded region about the x -axis?

- A. Cone B. Cylinder C. Pyramid D. Sphere E. N.O.T.

19) For which x -value below is the function $f(x) = \frac{x}{\sqrt{3} + \tan x}$ undefined?

- A. $\frac{5\pi}{6}$ B. $\frac{2\pi}{3}$ C. $\frac{4\pi}{3}$ D. $\frac{7\pi}{6}$ E. N.O.T.

- 20) Which of the following is the remainder after computing the polynomial division $\frac{8x^2-6x+1}{2x-3}$?
- A. -8 B. 10 C. 1 D. -9 E. N.O.T.

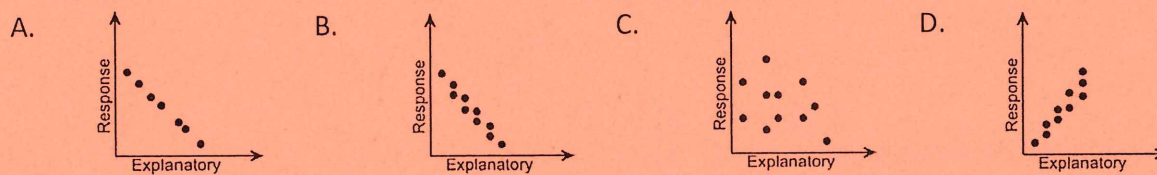
21) Solve the absolute value equation $\left| \frac{1}{7} - \frac{9}{14}x \right| + 1 = 6$.

- A. $x = \frac{-68}{9}$ or -8 B. $x = \frac{68}{9}$ or 8
 C. $x = 8$ or $\frac{74}{9}$ D. $x = -8$ or $\frac{-74}{9}$ E. N.O.T.

22) Solve the following equation: $|x-3| = |3-x|$.

- A. 3 B. 7 C. No solution D. All real numbers E. N.O.T.

23) Match the linear correlation coefficient to the scatter diagram. $r = -0.025$

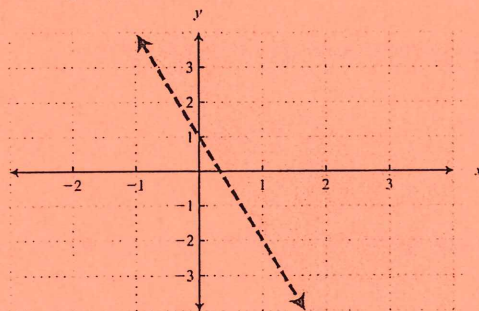


E. N.O.T.

24) Which of the following functions represent the sequence: 0, 3, 8, 15, 24, 35, ...

- A. $f(n) = 3n, n \geq 0$ B. $f(n) = 3n - 1, n \geq 1$
 C. $f(n) = n^2 - 1, n \geq 1$ D. $f(n) = 3 - n^3, n \geq 0$ E. N.O.T.

25) Which inequality is shown in the graph below?



- A. $x - y > 3$ B. $y \leq -2x + 5$ C. $y > -3x + 1$ D. $y > -3x$ E. N.O.T.

26) Divide $2x^3 + 4x^2 - x + 5$ by $x + 2$.

A. $2x^2 - 1 + \frac{7}{x+2}$

B. $2x^2 + 1 + \frac{7}{x+2}$

C. $2x - 1 + \frac{7}{x+2}$

D. $x^2 - 1 + \frac{7}{x+2}$

E. N.O.T.

27) Solve $x^3 + 3x^2 - 4x = 0$.

A. $x = 0, -4, \text{ or } 3$

B. $x = 0, -4, \text{ or } 1$

C. $x = 2, -4, \text{ or } 1$

D. $x = 0, -4 + i, \text{ or } -4 - i$

E. N.O.T.

28) Solve $\ln(x) + \ln(2) = 4$.

A. $\frac{e^2}{4}$

B. $\frac{e^4}{2}$

C. $e^x = 4$

D. 2

E. N.O.T.

29) The value of the determinant of $\begin{bmatrix} 2 & -2 & 2 \\ 3 & -3 & 3 \\ 4 & -4 & 4 \end{bmatrix}$ is

A. 9

B. -9

C. 3

D. 0

E. N.O.T.

30) Find the total sum of $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$

A. $\frac{2}{3}$

B. $\frac{3}{2}$

C. $\frac{5}{3}$

D. $\frac{3}{5}$

E. N.O.T.

31) Find the vertex of the following function: $g(x) = 2x^2 - 20x - 3$.

A. (0, -9)

B. (5, -53)

C. (-5, 23)

D. (-53, 5)

E. N.O.T.

32) Find the radius of the following circle: $x^2 + y^2 + 8x - 2y + 15 = 0$.

A. 1

B. 2

C. $\sqrt{3}$

D. $\sqrt{2}$

E. N.O.T.

33) Solve the linear system:

$$2x + y + 2z = 10$$

$$x + 2z = 5$$

$$x - 2y + 2z = 1$$

A. $(x, y, z) = (1, 2, 3)$

B. $(x, y, z) = (3, 2, 1)$

C. $(x, y, z) = (0, 1, 5)$

D. $(x, y, z) = (2, 1, 3)$

E. N.O.T.

34) Find the product AB for the given matrices.

$$A = \begin{bmatrix} 10 & 7 \\ 11 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 60 \\ 80 \end{bmatrix}$$

A. $\begin{bmatrix} 400 \\ -770 \end{bmatrix}$

B. $\begin{bmatrix} -680 \\ 770 \end{bmatrix}$

C. $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

D. $\begin{bmatrix} 1160 \\ 980 \end{bmatrix}$

E. N.O.T.

35) Evaluate the following:

$$\sum_{k=3}^6 (2k - 5)$$

A. 15

B. 12

C. 17

D. 16.5

E. N.O.T.

36) Evaluate $C(7,3)$.

A. 30

B. 35

C. 40

D. 30

E. N.O.T.

37) Find A^{-1} for $A = \begin{bmatrix} 1 & 4 \\ 2 & 9 \end{bmatrix}$.

A. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 9 & 4 \\ 2 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 9 & -4 \\ -2 & 1 \end{bmatrix}$

D. $\begin{bmatrix} -9 & 1 \\ 1 & 2 \end{bmatrix}$

E. N.O.T.

38) Which of the following is NOT a measure of the center for a data set?

- A. mode B. median C. census D. mean E. N.O.T.

39) Find the volume of a circular cylinder that has a radius of 3 inches and a height of 2.5 feet. Assume all answers represent cubic inches.

- A. 19π B. 22.5π C. 225π D. 170π E. N.O.T.

40) Simplify $\frac{x^2-1}{x^2+x-6} \div \frac{x-1}{x+3}$.

- A. $\frac{x+1}{x-2}$ B. $\frac{3x+1}{x-2}$ C. 0 D. $\frac{-3x-1}{x+2}$ E. N.O.T.